HEAT EXCHANGE IN A DRAINING LIQUID FILM IN THE INITIAL THERMAL PART

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An approximate solution of the heat-transfer problem in a draining liquid film with a parabolic velocity profile and boundary conditions of the second kind is given.

The small thickness of draining liquid films and their large surface area considerably facilitates thermal, diffusion, and other processes. The processes occurring in draining liquid films therefore have a number of technical applications and have recently been given considerable attention.

Considerable mathematical difficulties arise in the analytical solution of heat-transfer problems in the initial thermal part (even assuming a plane film surface), and up till now only the approximate Nusselt solution [1] and the solution obtained in [2] for a constant wall temperature are known. A drawback of the solution obtained in [2] is the fact that it is presented in the form of an infinite series which converges slowly for large Peclet numbers and small values of the dimensionless longitudinal coordinate.

Below we consider heat transfer in the initial thermal part for a laminar draining film along the vertical surface and for boundary conditions of the second kind. The flow is assumed to be hydrodynamically stabilized, the velocity profile is assumed to be parabolic, and the physical properties of the liquid are assumed to be constant.

In the boundary-layer approximation the equation of convective heat transfer with the corresponding initial and boundary conditions has the form

$$(2\eta-\eta^2)\frac{\partial\Theta}{\partial\xi}=\frac{\partial^2\Theta}{\partial\eta^2}, \tag{1}$$

$$\Theta(0,\ \eta)=0,\ \frac{\partial\Theta(\xi,\ 0)}{\partial\eta}=1,\ \ \frac{\partial\Theta(\xi,\ 1)}{\partial\eta}=0. \tag{2}$$

A numerical solution of problem (1), (2) was obtained in [3].

1. Method of Solution. The essence of the approximate method used to solve problem (1), (2) reduces to approximating the temperature profile by a function with unknown generalized coordinates. This approach recommends itself for solving nonstationary one-dimensional heat-conduction problems [4, 5]. However, unlike the methods in the heat-conduction theory, the profile parameter is not specified arbitrarily [5] nor is it determined from the condition for a certain functional to be stationary [6], but is found directly from the differential equations. The latter enables the accuracy of the solution to be increased considerably.

We will distinguish two stages of the heat-transfer process. In the first a thermal boundary layer develops and the temperature profile will be sought in the form

$$\Theta=\Theta_1\left[1-\frac{\eta}{q(\xi)}\right]^{n_1}. \tag{3}$$

The first stage is concluded when the thickness of the thermal boundary layer reaches $q=1$. In the second, there is a change in the temperature on the free surface of the film, and the temperature profile is sought in the form

$$\Theta=(\Theta_1-\Theta_2)(1-\eta)^{n_2}+\Theta_2. \tag{4}$$

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TABLE 1. Comparison of the Average Nusselt Numbers <Nu>

ξ	$6,666\cdot10^{-3}$	0,02	0,05	0,1	0,5	1	2	∞
Solution								
Ours	6,960	4,829	3,596	2,770	2,220	2,152	2,110	2,083
Numerical [3]	6,60	4,79	3,43	2,83	2,15	2,08	2,03	—

Hence, to determine the unknown functions q and Θ_2, and the parameters n_1, n_2, it is necessary to have two equations at each stage of the heat transfer.

2. First Stage of the Process. We have directly from Fourier's law and (3)

$$\Theta_1 = q/n_1. \tag{5}$$

Substituting (3) into (1), taking (5) into account, and integrating with respect to η within the limits of the boundary layer, we obtain

$$1 = \frac{2}{n_1(n_1+1)(n_1+2)} \frac{d}{d\xi}\left(q^3 - \frac{1}{n_1+3} q^4\right). \tag{6}$$

We multiply (1) by Θ and integrate the latter equation taking (3) and (5) into account within the same limits. After appropriate transformations we obtain the second equation

$$\frac{n_1(n_1-1)}{2n_1-1} q = \frac{1}{(2n_1+1)(2n_1+2)} \frac{d}{d\xi}\left(q^4 - \frac{1}{2n_1+3} q^5\right). \tag{7}$$

Integrating (6) and (7) we have

$$\frac{n_1(n_1+1)(n_1+2)}{2} \xi = q^3 - \frac{1}{n_1+3} q^4, \tag{8}$$

$$\frac{n_1(n_1-1)(2n_1+1)(2n_1+2)}{2n_1-1} \xi = \frac{4}{3} q^3 - \frac{5}{4(2n_1+3)} q^4. \tag{9}$$

We expand (8) and (9) in terms of ξ and equate. Assuming $q = 1$ in the equations obtained, we finally obtain the following connection equation:

$$128n_1^4 + 94n_1^3 - 343n_1^2 - 354n_1 - 45 = 0$$

with the positive root $n_1 = 1.7552$.

The implicit relations (8) and (9) for q are inconvenient for practical use. If we assume that the maximum values of the coefficient of q^4 on the right sides of (8) and (9) do not exceed 21 and 14%, respectively, of the coefficient of q^3, an explicit relationship for q can be obtained by the perturbation method, assuming these coefficients to be small parameters. In this case we obtain a more accurate relationship from (9). Confining ourselves to the linear term with respect to the small parameter, we obtain from (9)

$$q = \left(\frac{3}{4} A\xi\right)^{1/3} + \frac{5}{16(2n_1+3)} \left(\frac{3}{4} A\xi\right)^{2/3},$$
$$A = \frac{n_1(n_1-1)(2n_1+1)(2n_1+2)}{2n_1-1}. \tag{10}$$

Comparison with the accurate implicit relationship (9) shows that the maximum error in determining q from (10) is 0.7%. It follows from (8) and (9) that the extent of the first stage of the process $\xi_1 = 0.087$.

3. Second Stage of the Process. Substituting (4) into (1) and integrating, we obtain

$$1 = \frac{d}{d\xi}\left[(\Theta_1 - \Theta_2) \frac{2}{(n_2+1)(n_2+3)} + \frac{2}{3} \Theta_2\right]. \tag{11}$$

Multiplying (1) by Θ and integrating, taking (4) into account, we obtain the second equation

$$\Theta_2 - \frac{n_2(\Theta_1 - \Theta_2)}{2n_2-1} = \frac{d}{d\xi}\left[\frac{(\Theta_1 - \Theta_2)^2}{(2n_2+1)(2n_2+3)} + \frac{2\Theta_2(\Theta_1 - \Theta_2)}{(n_2+1)(n_2+3)} + \frac{1}{3}\Theta_2^2\right]. \tag{12}$$

Comparing (11) and (12), taking into account the equation $n_2(\Theta_1 - \Theta_2) = 1$, we obtain the equation

$$\left[\frac{\Theta_2'}{n_2(n_2+1)(n_2+3)} + \frac{1}{3}\Theta_2\Theta_2'\right] \Big/ \left(\Theta_1 - \frac{1}{2n_2-1}\right) = \frac{1}{3}\Theta_2' . \tag{13}$$

From (13) we obtain the connection equation

$$n_2^2 + 3n_2 - 7 = 0 \tag{14}$$

with the positive root $n_2 = 1.5414$.

Integrating (11), we obtain

$$\Theta_2 = \frac{3}{2}(\xi - \xi_1). \tag{15}$$

Taking (15) into account, the temperature profile (4) takes the form

$$\Theta = \frac{1}{n_2}(1-\eta)^{n_2} + \frac{3}{2}(\xi - \xi_1). \tag{16}$$

4. Comparison with the Numerical Solution. To compare the solution obtained with the well-known numerical solution [3], we will find the local and average Nusselt numbers. The local Nu numbers can be found from the relation

$$\mathrm{Nu} = \frac{1}{\Theta_1 - \Theta_{av}} . \tag{17}$$

Calculating the average temperature over the section of the film, we have from (17) for the first and second stages of the process, respectively,

$$\mathrm{Nu} = n_1/q\left[1 - \frac{3}{(n_1+1)(n_1+2)}q^2 + \frac{3}{(n_1+1)(n_1+2)(n_1+3)}q^3\right]^{-1}, \tag{18}$$

$$\mathrm{Nu} = \langle \mathrm{Nu} \rangle = n_2 \Big/ \left[1 - \frac{3}{(n_2+1)(n_2+3)}\right] = 2.083. \tag{19}$$

The average <Nu> numbers in the first stage of the process can be calculated from

$$\langle \mathrm{Nu} \rangle = \frac{1}{\xi}\int_0^{\xi} \mathrm{Nu}\, d\xi. \tag{20}$$

Evaluation of the integral in (20) gives the relation

$$\langle \mathrm{Nu} \rangle = \frac{1}{\xi}[3.3053 - 0.9626\ln(0.6228q+1) - 0.8520\ln(0.0979q^2 - 0.6228q + 1) - 2.2447\,\mathrm{arctg}(10.1446 - 3.1897q)]. \tag{21}$$

The results of a comparison of the average <Nu>, calculated from Eqs. (19) and (21) and from the data given in [3], are given in Table 1. In the calculations using (21), q was calculated from Eq. (10). As can be seen from Table 1, there is good agreement between the data over the whole range of ξ, which confirms the accuracy of the solution obtained. We also note that in [7] for stabilized heat transfer a value <Nu> = 2.063 was obtained.

As shown in [3], the experimental values of <Nu> from some experiments are ≈10–20% greater than the theoretical values. This is usually explained by the effect of waviness on the free surface [8]. This is undoubtedly confirmed in [3]. The variation in the physical properties of the liquid, primarily the viscosity, is obviously partly responsible for the difference between the experimental and theoretical results.

In conclusion, we note that the solution obtained holds for moderate temperature gradients. For considerable gradients, the heat transfer begins to depend very much on the value of heat flow. This problem requires special consideration.

NOTATION

$\Theta = (t - t_0)/(H\delta/\lambda)$, dimensionless temperature; t_0, temperature of the liquid at the entrance; Θ_1, Θ_2, dimensionless temperature of the wall and free surface; Θ_{av}, dimensionless average temperature of the liquid over the transverse cross section of the film; H, density of the heat flow on the wall; $\xi = (x/\delta)\mathrm{Pe}$, $\eta = y/\delta$, dimensionless coordinates; δ, thickness of the film; $\mathrm{Pe} = u_0\delta/a$, Peclet criterion; u_0, velocity on the free surface; λ, a, thermal conductivity and thermal diffusivity of the liquid; $q(\xi)$, thickness of the thermal boundary layer; n_1, n_2, parameters of the temperature profile in the first and second stages of the process; $\mathrm{Nu} = \alpha\delta/\lambda$, $\langle\mathrm{Nu}\rangle = \langle\alpha\rangle\delta/\lambda$, local and average Nusselt numbers; α, $\langle\alpha\rangle$, local and average heat-transfer coefficients.

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REFINED METHOD OF CALCULATING HEAT EXCHANGE IN THE CONDENSATION OF STATIONARY STEAM ON FINNED HORIZONTAL TUBES

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A relation is obtained for calculating the heat-transfer rate in the condensation of pure vapors on finned tubes with allowance for fin efficiency.

The study of the heat-exchange laws in the condensation of vapors of liquids is directed toward solving important practical problems of finding surfaces on which heat- and mass-transfer processes can occur efficiently. A certain amount of experimental material has already been accumulated in this direction, including data on heat exchange in the condensation of vapors of Freons 11, 12, 22, and 113, and water on single horizontal tubes with transverse finning. These tubes have been made of different materials and have had fins of various geometries. The main data on the test conditions and geometrical characteristics of these tubes, including the findings in [1, 2], are presented in Table 1. These studies have proposed relations in the form of criterial equations, including equations of the Nusselt type for smooth tubes, with the introduction of constant coefficients to account for the specifics of heat exchange on the fins. Analysis of these relations shows that they do not allow for generalization of the data of various authors which is in the literature. This is due to the complexity of the process of condensation on finned tubes. In contrast to the same process on smooth tubes, here condensation is determined by several new factors: the geometry of the surface, the heat conductivity of the wall material, and, as noted in certain investigations, the effect of surface tension.

Evidently, the only way out of this dilemma is to obtain theoretical solutions and refine these solutions on the basis of experimental data.

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